The Tabulation of Symmetric Functions.

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M. Faà de Bruno gives, in his *Théorie des Formes Binaires*, tables of the values of symmetric functions, which are symmetrically arranged and at the same time confined to a half square. In these tables the functions of the self-conjugate partitions are placed in the middle. The remainder of the functions are separated into pairs, a pair consisting of a function and the function of its conjugate partition, and the members of each pair are disposed symmetrically about the middle. How he obtains a suitable arrangement of these pairs he does not say. My object is to show that such an arrangement is always possible, and at the same time to indicate how it can be obtained.

I shall first show that such an arrangement is possible in the tables giving the values of the combinations of the simple symmetric functions in terms of the general symmetric functions.

Let a_1, a_2, etc. denote the partitions of any number arranged in natural or dictionary order; P_1, P_2, etc. the combinations, and ϕ_1, ϕ_2, etc. the symmetric functions corresponding to these partitions. I shall represent the conjugate of the partition a_λ by $a_{\lambda'}$, and the coefficient of ϕ_μ in the value of P_λ by $(\lambda\mu)$.

Professor Cayley has shown (*Phil. Trans.* 1857) that $(\lambda\mu) = 0$, if a_μ is prior to $a_{\lambda'}$, the conjugate of a_λ. We have then

$$(\lambda\mu) = 0 \quad \mu < \lambda'. \tag{1}$$

M. Betti proved (*Tortolini*, 1858)

$$(\lambda\mu) = (\mu\lambda)$$

therefore

$$(\lambda\mu) = 0 \quad \lambda < \mu'. \tag{2}$$

If a_λ is a self-conjugate partition, $\lambda = \lambda'$ and

$$(\lambda\mu) = 0 \quad \begin{matrix} \mu < \lambda \\ \lambda < \mu', \end{matrix} \tag{3}$$

if in addition a_μ is a self-conjugate partition

$$(\lambda\mu) = 0 \quad \begin{matrix} \mu < \lambda \\ \lambda < \mu. \end{matrix} \tag{4}$$

	41^8	3^4	3^321	3^22^3	3^221	3^31^3	$3^22^21^2$	32^41	321^7	32^21^5	3^21^6	31^9	2^6	2^51^2	2^41^4	2^31^6	2^21^8	21^{10}	1^{12}
41^8	-12	$+3$	-48	-24	$+40$	$+180$	$+60$	-180	-240	$+42$	$+252$	-96	$+12$	$+2$	-36	$+105$	-112	$+54$	-12
3^4	$+1$	-3	$+37$	$+24$	-18	-114	-49	$+70$	$+130$	-9	-87	$+19$	-1	-2	$+25$	-50	$+35$	-10	$\mathbf{1}$
3^321	$+2$	-3	-42	-6	$+28$	-45	-30	$+20$	$+70$	-17	-32	$+16$	-2	$+2$	-16	$+20$	-8	$\mathbf{1}$	10
3^22^3	$+3$	$+6$	-42	-3	-42	-30	-6	-21	-36	-6	$+63$	$+17$	-3	-2	$+9$	-6	$\mathbf{1}$	8	66
3^221^2	-3	$+37$	-3	-42	-28	$+8$	$+8$	$+16$	-4	-4	-28	$+8$	$+8$	-4	$+2$	$\mathbf{1}$	6	15	220
3^31^3	-3	-12	-11	$+13$	$+16$	-5	-5	-11	-5	$+13$	$+10$	-5	-5	$+5$	$\mathbf{1}$	4	15	20	495
2^6	$+3$	-12	-2	$+6$	$+9$	-2	-2	-12	-6	$+9$	$+6$	-2	$+5$	$\mathbf{1}$	6	20	70	56	792
2^51^2	-1	$+3$	-27	$+3$	$+3$	-9	-27	$+18$	$+54$	-9	-30	$+9$	$\mathbf{1}$	3	15	70	21	210	924
2^41^4	-3	-3	$+19$	-3	-6	$+9$	$+18$	-7	-30	$+13$	$+14$	-7	$\mathbf{1}$	9	50	30	145	252	132
2^31^6	-3	$+3$	-13	-5	-11	$+5$	$+13$	$+10$	$+5$	-11	-5	$\mathbf{1}$	27	72	15	84	36	128	525
2^21^8	-3	$+6$	-12	-2	-9	$+6$	$+3$	$+3$	-18	$\mathbf{1}$	42	6	201	21	5	93	336	810	660
321^7	-3	$+7$	-3	-1	$+2$	$+7$	-3	$\mathbf{1}$	52	10	207	17	70	6	252	52	736	2460	1980
32^21^5	-3	-4	-1	$+2$	-1	$+7$	$\mathbf{1}$	165	17	6	126	35	10	3	532	165	1732	5544	2970
3^21^6	-3	-2	-2	$+4$	-3	$\mathbf{1}$	498	50	15	4	504	161	50	15	15	1526	4620	13860	3960
32^31^3	$+3$	-3	$+3$	$\mathbf{1}$	707	228	70	20	6	756	252	81	30	24	9	2128	6300	18480	7920
$3^2 21^4$	$+3$	-3	$\mathbf{1}$	660	228	81	30	9	630	210	70	20	24	6	9	1932	5670	16632	5544
$3^3 1^3$	-3	$\mathbf{1}$	1266	456	165	60	22	1260	455	165	60	22	8	3	3	3528	9870	27720	13860
3^4	$\mathbf{1}$	1710	639	240	90	36	1680	630	240	90	93	36	15	6	6	4620	12600	34650	18480
41^8	8	54	6	28	3	189	38	5	2	300	1665	418	87	12	1320	5940	16632
421^6	28	108	20	624	168	40	12	7	2	1528	5040	432	108	20	15840
42^21^4	56	168	30	1080	302	72	32	12	5	2392	7740	681	168	30	23760
4321^3	56	336	105	30	1218	378	115	30	34	8	9	2	3164	9450	1041	336	105	30	27720
$43 1^5$	165	760	230	60	3024	1008	320	132	93	37	22	8	3	6888	19740	2337	760	230	55440
$4^2 1^4$	360	1206	360	90	5040	1722	552	262	159	72	36	15	6	10752	30240	3696	1206	360	83160

3^31^3	3^22^3	3^321	3^4	41^8	421^6	42^21^4	431^5	42^21^2	431^3	421^4	42^4	432^21	43^21^2	4^221^2	43^22	51^7	521^5	52^21^3	531^4	52^31
+40	−24	−48	+3	−12	+84	−180	−72	+120	+240	+30	−12	−144	−72	+72	+36	+12	−72	+120	+60	−48
−18	+24	+37	−3	+1	−18	+70	+17	−76	−108	−8	+12	+111	+39	+39	−36	−1	+17	−54	−16	+37
−30	−6	+28	−3	+2	−14	+20	+32	+10	−100	−15	−8	+24	+62	+32	−16	−2	+12	−10	−30	−12
+17	−3	−42	+6	+3	−21	+45	+18	−39	−33	−21	+12	+9	−9	+45	+18	−3	+18	−30	−15	+21
+8	+8	+16	−3		+4	−20	−8	+24	+48	+6	−12	−16	−40	−40	−4	−4		−40	−20	+16
−5	−11	+13	−3			+5	−5	−15	+5	+5	+12	+4	+2	+2	−1		−5	+20	+10	−22
−2	+6	−12	+3					+6	−12	+3	−6	0	+18	0	0			−6	+6	+12
+18	−9	−27	+3	−1	+8	−20	−17	+16	+68	+8	−2	−51	−39	−24	+26	+1	−7	+14	+16	−7
−6	+9	+19	−3	−3	+23	−54	−22	+39	+73	+23	−4	−36	−20	−62	−2	+3	−20	+37	+19	−16
−11	−5	+13	−3	−1	+5	+5	−5	−27	−3	−3	0	+16	+34	+16	−14	+4	−27	+15	+23	−26
+6	−2	−12	+3	−2	+12	0	−18	−12	−1	+4	+24	−6	+16	−6	+4	+2	−12	+18	+14	−4
+3	+3	−18	+6		−1		+3	−17	−5	−5	−20	+15	+15	−4	+5	+5	+1	−4	−6	+3
−1	+2	+7	−3		−2		+1	−5	+7	+1	−8	+15	−4	−7	+1	−7	+5	−23	−4	+19
+2	−1	+7	−3				−1	+7	+1	−3			−7	+5	+1	+5	+1	+1	−6	−1
−2	−2	+4	−3					−3	+4	−3					0	+8	+8	+2	+2	−4
1		−3	+3				−3	+3	+3	−3	+9		−9	−9		+3	+3	+3	−3	−9
	1	−3	+3					−2	+6	+3	−7		−9	−9		+5	+11			+2
3	3	**1**	−3					−3	−1	+5	+11		−4							+3
6	6	3	**1**										−4							
				1	−8	+20	+8	−16	−32	−8	+2	+24	+12	+24	−8	−1	+7	−14	−7	+7
				8	**1**	−6	−1	+9	+11	+1	−2	−17	−5	−10	+8	−4	+27	−51	−27	+23
				28	6	**1**	−2	−4	+8	+2	+2	0	−9	−6	+2		−1	+4	+3	−2
				56	13	2	**1**		−5	−1	+5	+5	+4	−5			−3	+15	+2	−15
				56	15	4		**1**	−3	+3	−2	+6	+3	−9	−7			−1	+2	+2
				168	51	14	5	3	**1**	−3	−3	−1	+11	+5				−3	+4	+9
				336	108	30	16	6	3	**1**	−4						−4			−2

531^5	52^21^3	531^4	52^31	61^6	621^4	4^22^2	5321^2	71^5	541^3	62^21^2	631^3	731^3	81^4	4^231	532^2	53^21	5421	5^21^2	63^2	6321	641^2	72^21	731^2
−72	+120	+60	−48	−12	+60	+18	−144	+12	−48	−72	−48	−48	−12	+86	+86	+86	+72	+18	+12	+72	+86	+86	+86
+17	−54	−16	+37	+1	−16	−18	+78	−1	+15	+39	+15	+15	+1	−25	−36	−25	−50	−7	−12	−50	−14	−25	−14
+12	−10	−30	−12	+2	−10	+2	+64	−2	+28	+2	+28	+8	+2	−36	+4	−36	−32	−13	+8	−32	−26	+4	−26
+18	−30	−15	+21	+3	−15	−18	+9	−3	+89	+18	+18	+12	+12	−9	−9	+18	−45	−18	−12	+9	−36	−9	−9
+24	−40	−20	+16	+4	−20	+22	+48	−4	+16	+24	+16	+16	+4	+44	−20	−4	−8	−18	+4	−40	−4	−20	−4
−5	+20	+10	−22	+5	−25	−18	−31	−5	−22	+37	+13	+13	+12	−1	+34	−1	+33	+17	−12	−37	−1	−1	−1
−7	−6	+6	+12	−1	+6	+9	0	−6	−12	−18	−12	+24	+6	−18	−18	0	0	+9	+6	+36	+18	−18	−18
−20	+14	+16	−7	−3	+6	+8	−48	+1	−15	−9	−15	−5	−1	+25	+16	+25	+30	+7	+2	+30	+14	+5	+14
−27	+37	+19	−16	−4	+17	+16	−43	+3	−43	−23	−16	−14	−3	+34	+5	+7	+73	+20	+4	+19	+40	+12	+13
−12	+15	+23	−26	−2	+23	−4	−63	+4	−22	−33	−19	−19	−4	−34	+29	−1	+31	+25	+8	+33	+18	+18	+15
+1	+18	+14	−4	−5	+10	−14	−32	+2	−12	−10	−12	−8	−2	−4	+4	+20	0	+13	0	+16	+10	+4	+10
+5	−4	−6	+3	−5	+29	−4	+21	+5	+11	−42	−24	−24	−5	−26	−14	−6	−19	−10	+8	+55	+11	+24	+11
	−28	−4	+19		+25	+16	+39	+5	+3	−27	−26	−20	−5	+10	−23	−17	−28	−4	+4	+30	+27	+12	+21
	+1	−6	−1		−1	0	+7	+6	+19	+2	+14	−28	−13	+26	+2	+5	−25	−16	0	−27	−27	+26	+15
	+2	+2	−4		−6	−12	−16	+6	+4	+22	+4	−24	−6	−8	+16	+4	+16	−5	−12	−24	−18	+16	+30
	+3	−3	−9		−3	0	+9	+3	−3	+9	+6	−12	−3	+9	+9	−9	+9	0	0	−27	0	+9	+9
+7	−14	−7	+7	+1	−6	−8	+21	−1	+15	+9	+6	+5	+1	−16	−7	−7	−30	−7	−2	−12	−14	−5	−5
+27	−51	−27	+23	+4	−23	+8	+75	−4	+26	+32	+28	+19	+4	+9	−21	−24	−39	−27	−6	−41	−22	−17	−19
−1	+4	+3	−2	+5	−29	−4	−15	−5	−5	+42	+27	+24	+5	+16	+4	+11	+16	+3	−10	−49	−25	−23	−22
−8	+15	+2	−15	+5	−27	−2	−23	−5	−1	+33	+28	+22	+5	−9	+19	+8	+18	+2	−4	−42	−29	−16	−23
	−1	+2	+2		+1	+8	−3	−6	−8	−3	−3	+29	+6	+1	−2	+2	+7	+9	+2	+5	+17	−29	−25
	−3	+4	+9		+7	−4	−11	−12	+2	−25	−11	+58	+12	−10	−11	+14	−10	+1	+8	+56	+5	−42	−49
			−2		+2	+2	+8	−2	0	−8	0	+8	+2	+4	−4	−8	0	−1	+4	+8	+2	−4	−10

	$543\,1$	51^2	62^3	6321	$64\,1^2$	$72^2 1$	$73\,1^2$	$82\,1^2$	91^3	4^3	543	$5^2 2$	63^2	642	651	732	741	82^2	831	921	101^2	6^2	75	84	93	102	111	12
12	+72	+18	+12	+72	+36	+86	+86	+86	+12	−4	−24	−12	−12	−24	−24	−24	−24	−24	−12	−24	−24	−12	+6	+12	+12	+12	+12	−12
111	−50	−7	−12	−50	−14	−25	−14	−14	−1	+4	+24	+12	+12	+24	+13	+24	+13	+12	+13	+18	+1	−6	−6	−12	−12	−12	−1	+12
102	−32	−13	+8	−32	−26	+4	−26	−6	−2	+4	+24	+24	+2	+12	+4	+24	+4	+24	−8	+24	+4	+2	−6	−12	−12	−12	+8	−12
93	−45	−18	−12	−36	−9	−9	−9	−3	+4	+12	−15	+24	+24	−3	+24	+12	−3	+24	+12	−3	+12	−12	−12	+15	−12	−12	−12	+12
84	−8	−18	+4	−40	−4	−20	−4	−4	−12	−12	−8	+12	+12	−8	+24	+24	−8	−6	−12	+20	−12	−12	−12	+12	−12	−12	−12	+12
75	+33	+17	−12	−37	−1	−1	−1	−36	−12	+4	−11	−23	+12	+24	−11	−11	−11	+12	+24	+24	+12	−6	−6	−12	−12	−12	+12	+12
6^2	0	+9	+6	+36	+18	−18	−18	−18	−6	+3	+12	+6	−12	−24	−24	+12	+12	+6	+15	−6	−6	+6	+6	+6	+6	+6	+6	+6
101^2	+30	+7	+2	+30	+14	+5	+14	+4	+1	−4	−24	−7	−12	−14	−13	−14	−13	−2	−13	−3	−1	+6	+12	+12	+1	+1	−12	−12
921	+73	+20	+4	+19	+40	+12	+13	+11	+3	−8	−21	−14	+3	−28	−37	−1	−37	−4	−10	−8	−3	+12	+24	+24	−3	+4	+13	−24
831	+31	+25	+8	+33	+18	+18	+15	+15	+4	+8	+11	−24	+3	−16	−37	−21	−5	−8	−2	−10	−13	+12	+24	−8	−3	+24	+13	−24
82^2	0	+13	0	+16	+10	+10	+6	+4	+4	−8	−2	−12	−24	−4	−8	−8	−4	−2	−8	−4	+6	+12	−4	+12	−8	+12	−12	−12
741	−19	−10	+8	+55	+11	+24	+11	+18	+18	+8	+19	+11	−24	−16	−2	−13	+2	−8	−5	−37	−13	+12	−11	−8	+24	+24	+13	−24
732	−28	−4	+4	+30	+27	+12	+21	+15	+5	−8	+14	+21	+3	−28	−13	−8	−13	−4	−21	−1	−14	+12	−11	+24	−3	+4	+24	−24
651	−25	−16	0	−27	−27	+26	+15	+50	+13	−8	−13	+11	+12	+24	+40	−13	−2	−24	−37	−37	−13	−24	−11	+24	+24	+24	+13	−24
642	+16	−5	−12	−24	−18	+16	+30	+10	+14	+8	−16	−14	+12	+28	+24	−28	−16	+12	−16	−28	−14	−24	+24	−8	+24	+4	+24	−24
63^2	+9	0	0	−27	0	+9	+9	+9	+3	−4	+3	−12	+6	+12	+12	+3	−24	−12	+3	+3	−12	−12	+12	+12	−15	+12	+12	−12
$5^2 2$	−3	−7	+2	+17	−4	−19	−4	+21	+7	−4	+11	+3	−12	−14	+11	+21	+11	−2	−24	−14	−7	+6	−23	+12	+12	+2	+12	−12
543	+8	+1	+8	−4	+5	−6	−22	+13	+15	+8	−14	+11	+3	−13	+14	+19	−8	+11	−21	−24	−11	−8	−3	+24	+24	−24	−24	−24
4^3	−8	+6	−4	+8	−4	+4	−4	−4	+4	−4	+8	−8	−8	+8	+4	+8	−8	−4	+2	+4	−12	+4	+4	+4	−4			−4
91^3	−30	−7	−2	−12	−14	−5	−5	−4	−1	+4	+15	+7	+3	+14	+13	+5	+13	+2	+4	+3	+1	−6	−12	−12	−3	−2	−1	+12
$82\,1^2$	−39	−27	−6	−41	−22	−17	−19	−15	−4	+13	+21	+9	+10	+50	+15	+18	+6	+15	+11	+4	−18	−36	−4	−9	−6	−14	+36	+88
$73\,1^2$	+16	+3	−10	−49	−25	−23	−22	−19	−5	−4	−22	−4	+9	+30	+15	+21	+11	+10	+15	+13	+14	−18	−1	−4	−9	−26	−14	+88
$72^2 1$	+18	+2	−4	−42	−29	−16	−28	−17	−5	+4	−6	−19	+9	+16	+26	+12	+24	+4	+18	+12	+5	−18	−1	−20	−9	+4	−25	+86
$64\,1^2$	+7	+9	+2	+5	+17	−29	−25	−22	−14	−4	+5	−4	0	−18	−27	+27	+11	+10	+18	+40	+14	+18	−1	−4	−36	−26	−14	+36
6321	−10	+1	+8	+56	+5	−42	−49	−41	−12	+8	−4	+17	−27	−24	−27	+30	+55	+16	+33	+19	+30	+86	−37	−40	+9	−32	−50	+72
62^3	0	−1	+4	+8	+2	−4	−10	−6	−2	−4	+8	+2	0	−12	0	+4	+8	+8	+4	+2	+6	−12	+4	+12	−12	+8	+12	−12

n	sequence of values
168	3, 8, 22, 37, 93, 132, 320, 1008, 3024, 60, 230, 760, 2337, 6888, 19740, 54440
336	6, 15, 36, 72, 159, 262, 552, 1722, 5040, 90, 360, 1206, 3696, 10752, 30240, 83160
70	2, 4, 20, 12, 54, 40, 160, 490, 1512, 66, 172, 480, 1380, 4004, 11592, 33364
250	3, 11, 9, 30, 83, 86, 228, 250, 635, 1771, 4914, 210, 565, 1536, 4188, 11396, 30870, 83160
420	7, 18, 31, 42, 96, 154, 357, 420, 976, 2632, 7056, 300, 820, 2220, 5952, 15848, 42000, 110880
640	12, 39, 42, 96, 216, 279, 606, 776, 1652, 4410, 11592, 480, 1320, 3552, 9417, 24696, 64260, 166320
890	18, 216, 96, 216, 238, 546, 600, 1390, 3570, 9240, 480, 1225, 3132, 8040, 20720, 53550, 138600
80	6, 12, 24, 94, 468, 24, 180, 870, 3420, 11880, 47520
204	12, 78, 166, 703, 2520, 60, 348, 1422, 4968, 15840
388	6, 18, 84, 208, 306, 732, 2289, 6888, 120, 480, 1596, 4890, 14280, 40320, 110880
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15120	90, 1080, 2026, 2160, 3768, 4900, 9054, 40002, 512400, 960, 529200, 1247400
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3				168	51	14	5	3	1	−3	−3	−1	+11	+5	−3	+4	+9
6				336	108	30	16	6	3	1		−4			−2
	2			70	20	6		2		1	−4	+2	+4	+4		−1
9	11	3		280	95	32	10	11	3		4	1	−2	−1	−1		−3
24	18	7		420	150	53	20	18	7		6	2	1	−2	−2
42	31	12		840	315	114	55	39	18	4	12	5	2	1
42	39	18	4	560	210	80	30	31	12		12	5	2	1
				33	4								1	−7	+14	+7	−7
				204	46	7	3					7	1	−5	−1	+5
6				588	174	47	15	10	3			21	5	1	−2	−3
15				1064	331	90	43	18	8	2		42	11	2	1
18	24	6		966	320	106	30	36	9	13	3	35	10	3	1
				600	135	20	10					36	5
96	68	27		2548	926	327	145	110	47	8	34	13	5	2	2	105	35	11	4	3
72	68	30	6	1120	440	172	80	68	30	6	28	12	4	2	2
36				2940	910	245	120	48	22	6						196	50	9	4
90				7560	2355	630	335	120	60	20						672	171	30	15
189	117	51		4536	1713	618	321	207	103	30	60	27	12	7		210	75	24	13	6
216	150	60		6888	2489	872	390	291	124	24	88	34	12	6		476	155	48	16	13
444	258	117		11844	4446	1593	828	528	264	78	150	68	31	18		882	306	96	48	24
1050	570	270		29400	11085	3970	2135	1305	675	220	360	170	80	50		2856	985	306	155	75
2320	1360	630		70560	26880	9660	5460	3150	1715	630	840	420	210	140		8232	2856	882	476	210
141	117	58	12	1680	690	284	140	117	58	12	48	24	12	5	5
162	153	66	12	3360	1280	488	210	189	78	12	76	31	10	4	5	140	50	18	6	7
318	264	132	28	4900	1950	781	360	314	147	24	126	60	29	10	12	210	80	31	12	12
582	467	237	48	8690	3575	1466	755	599	305	80	244	123	60	31	24	420	170	68	34	27
1080	828	438	90	15120	6420	2694	1500	1116	610	201	456	244	126	76	48	840	360	150	88	60
360	344	144	24	9030	3420	1294	560	498	204	36	201	80	24	12	12	630	220	78	24	30
13	10	5	1	22	91	37	19	15	7	2	6	3	1			17	6	2	1	1

−3	+4	+9		+7	−4	−11	−12	+2	−25	−11	+53	+12	−10	−11	+14	−10	+1	+8	+56	+5	−42	−49
	−2			+2	+2	+8	−2	0	−8	0	+8	+2	+4	−4	−8	0	−1	+4	+8	+2	−4	−10
		−1		−4	+3	−7	+1	−2	−1	+7	−8	+1	−5	+10	+7	−1	+1	+9	+2	+3		
		−3		0	+8	−2	+3	−4	−8	+14	+10	+3	−10	−10	+10	0	−10	+7	+18	+16		
				+4	−1	+5	+4	−7	−4	+7	0	+2	0	−10	−5	−8	+14	+2	+8	+11		
				−2	−2	−1	+2	+3	−7	+7	−1	+4	+2	+3	+1	−4	−11	−2	+4			
				−2	0						−1	−1	0	+10	−8	+4	−10	+1	−9	+16		
1	−7	+14	+7	−7	−1	+6	−21	+1	−7	−9	−6	−5	−1	+7	+7	+14	+7	+2	+12	+6	+5	+5
1	−5	−1	+5	−5	+29	+9	+5	+1	−41	−29	−24	−5	−7	−4	−8	−1	+8	+53	+29	+22	+24
5	1	−2	−3		−1	+6	+6	+2	+3	+3	−29	−6	+3	−7	−4	−2	0	−11	−3	+28	+27
11	2	1		−4	−4	+9	−1	+16	+3	−41	−9	+2	+4	+3	+1	−8	−25	−3	+33	+42
10	3	1		−3	+3	−1	+2	+1	−7	−1	+5	−2	−7	0	+2	−8	−1	−5
5			1	−6		−1	+9	+6	+5	+1			−2	−12	−6	−5	−5
35	11	4	3	1	−3	−4	+6	+9	−21	−2	−1	+8	+3	+8	−11	−3	−23	−15
				1		−2	−2	+4	0	−4	+2	−4	+8	−2	−4
50	9	4	6	1	−6	−4	−1	+29	+6			+2	+7	+1	−27	−29
171	30	15	37	6	1		−5	−1			+5	+5
75	24	13	6	3	1	−3	+5	−7	−3	−1	0	+9	+2	−15	−2
155	48	16	13	15	4	4	1	−2	−1	+7	−2	+4	+2	+2	+8
306	96	48	24	30	9	11	3	2	1	−5	+14	−3	−1	+15	+4
985	306	155	75	171	50	35	5	10	11	5	1	−7	−3	−1
2856	882	476	210	672	196	105	36	35	42	21	7	1
					2	0		1		−2	−1	+4	+5	−7	−5	+2
50	18	6	7		2	2		1	−2	−1	+4	−4	+11	−9	+4	−6
80	31	12	12	4	5		2	2	1	−2	+2	−1	+3	+5	−9
170	68	34	27	12	13	3	5	5	2	1	−4	−3	+6	+5	0
360	150	88	60	28	34	13	12	12	6	4	1	−2	+2
220	78	24	30	20	6	6	8	2	4	1	−3	+3	−1	+2
6?	2	1	1					2				1	−1

Partition	Coefficients
6321	$-10,\ +1,\ +8,\ +56,\ +5,\ -42,\ -49,\ -41,\ -12,\ +8,\ -4,\ +17,\ -27,\ -24,\ -27,\ +30,\ +55,\ +16,\ +33,\ +19,\ +30,\ +36,\ -37,\ -40,\ +9,\ -82,\ -50,\ +72$
63^3	$0,\ -1,\ +4,\ +8,\ +2,\ -4,\ -10,\ -6,\ -2,\ +8,\ +2,\ 0,\ -12,\ 0,\ +4,\ +8,\ 0,\ +8,\ +4,\ +2,\ +6,\ -12,\ +4,\ -12,\ +8,\ -12,\ +12$
521^2	$+10,\ +7,\ -1,\ +1,\ +9,\ +2,\ +3,\ -27,\ -7,\ +6,\ +1,\ -7,\ 0,\ -16,\ -4,\ -10,\ +13,\ +25,\ +20,\ +7,\ +9,\ +17,\ -18,\ -18,\ -13,\ -7,\ +18$
5421	$-10,\ +10,\ 0,\ -10,\ +7,\ +18,\ +16,\ -39,\ -30,\ -8,\ +8,\ -3,\ +9,\ +16,\ -25,\ -28,\ -19,\ 0,\ +31,\ +73,\ +30,\ 0,\ +33,\ -8,\ -45,\ -32,\ -50,\ +72$
5321^3	$-10,\ -5,\ -8,\ +14,\ +2,\ +8,\ +11,\ -24,\ -7,\ -4,\ +11,\ +1,\ -9,\ +4,\ +5,\ -17,\ -6,\ +20,\ -1,\ +7,\ +25,\ 0,\ -1,\ -4,\ +18,\ -36,\ -25,\ +86$
532^2	$+3,\ +1,\ -4,\ -11,\ -2,\ +19,\ +4,\ -21,\ -7,\ +4,\ -11,\ -4,\ +9,\ +16,\ +2,\ -23,\ -14,\ +4,\ +29,\ +5,\ +16,\ -18,\ +34,\ -20,\ -9,\ +4,\ -86,\ +36$
4^281	$+10,\ -8,\ +4,\ -10,\ +1,\ -9,\ +16,\ +9,\ -16,\ +4,\ -10,\ +1,\ +9,\ -8,\ +26,\ +10,\ -26,\ -4,\ -34,\ +34,\ +25,\ -18,\ -1,\ +44,\ -9,\ -86,\ -25,\ +86$
81^4	$+14,\ +7,\ +2,\ +12,\ +6,\ +5,\ +5,\ +4,\ +1,\ -7,\ -7,\ -3,\ -6,\ -18,\ -5,\ -5,\ -2,\ -4,\ -3,\ -1,\ +6,\ -12,\ -4,\ +3,\ +2,\ +1,\ -12$
721^3	$-8,\ -1,\ +8,\ +53,\ +29,\ +22,\ +24,\ +19,\ +5,\ +7,\ +7,\ -12,\ -24,\ -28,\ -20,\ -24,\ -8,\ -19,\ -14,\ -5,\ +24,\ +13,\ +16,\ +12,\ +15,\ -48$
631^3	$-4,\ -2,\ 0,\ -11,\ -3,\ +28,\ +27,\ +23,\ +6,\ +7,\ -3,\ +8,\ +4,\ +14,\ -26,\ -24,\ -12,\ -19,\ -16,\ -15,\ -12,\ +13,\ +16,\ +12,\ +28,\ +15,\ -48$
621^2	$+3,\ +1,\ -8,\ -25,\ -3,\ +33,\ +42,\ +32,\ +9,\ -7,\ -2,\ +9,\ +22,\ +2,\ -27,\ -42,\ -10,\ -33,\ -23,\ -9,\ -18,\ +37,\ +24,\ +18,\ +2,\ +39,\ -72$
5431	$-2,\ -7,\ 0,\ +2,\ -8,\ -1,\ -5,\ +26,\ +15,\ -5,\ +7,\ -3,\ +4,\ +19,\ +3,\ +11,\ -12,\ -22,\ -43,\ -15,\ -12,\ -22,\ +16,\ +39,\ +28,\ +15,\ -48$
71^5	$-2,\ -12,\ -6,\ -5,\ -5,\ -4,\ -1,\ +3,\ +6,\ +6,\ +5,\ +5,\ +2,\ +4,\ +3,\ +1,\ -6,\ -5,\ -4,\ -3,\ -2,\ -1,\ +12$
532^21	$+8,\ +3,\ +8,\ -11,\ -3,\ -23,\ -15,\ +75,\ +21,\ 0,\ -2,\ -4,\ +9,\ -16,\ +7,\ +39,\ +21,\ -32,\ -63,\ -43,\ -48,\ 0,\ -31,\ +48,\ +9,\ +64,\ +78,\ -144$
4^22^2	$0,\ -4,\ +2,\ -4,\ +8,\ -2,\ -4,\ +8,\ -8,\ +2,\ -4,\ +8,\ 0,\ -12,\ 0,\ +16,\ -4,\ -14,\ -4,\ +16,\ +8,\ +9,\ -18,\ +22,\ -18,\ +2,\ -18,\ +18$
621^4	$+2,\ +7,\ +1,\ -27,\ -29,\ -23,\ -6,\ -3,\ -6,\ -1,\ +25,\ +29,\ +10,\ +23,\ +17,\ +6,\ +6,\ -25,\ -20,\ -15,\ -10,\ -16,\ +60$
61^6	$+5,\ +5,\ +4,\ +1,\ -5,\ -5,\ -2,\ -4,\ -3,\ -1,\ 0,\ +5,\ +4,\ +3,\ +2,\ +1,\ -12$
52^31	$-3,\ -1,\ 0,\ +9,\ +2,\ -15,\ -2,\ +23,\ +7,\ +3,\ +2,\ -9,\ -4,\ -1,\ +19,\ +3,\ -4,\ -26,\ -16,\ -7,\ +12,\ -22,\ +16,\ +21,\ -12,\ +37,\ -48$
531^4	$-2,\ +4,\ +2,\ +2,\ +3,\ -27,\ -7,\ -3,\ +2,\ -6,\ -4,\ -6,\ +14,\ +23,\ +19,\ +16,\ +6,\ +10,\ -20,\ -15,\ -30,\ -16,\ +60$
52^21^3	$-3,\ -1,\ +15,\ +4,\ -51,\ -14,\ +3,\ +2,\ +1,\ -23,\ -4,\ +18,\ +52,\ +37,\ +14,\ -6,\ +20,\ -40,\ -30,\ -10,\ -54,\ +120$
521^5	$-3,\ -1,\ +27,\ +7,\ +5,\ +1,\ -12,\ -27,\ -20,\ -7,\ -5,\ +24,\ +18,\ +12,\ +17,\ -72$
51^7	$-4,\ -1,\ +2,\ +4,\ +3,\ +1,\ -4,\ -3,\ -2,\ -1,\ +12$
43^22^2	$-1,\ +4,\ +5,\ -7,\ -5,\ +2,\ +8,\ -8,\ -4,\ +11,\ -9,\ -9,\ +8,\ +1,\ +5,\ +5,\ 0,\ -14,\ -2,\ +26,\ -1,\ -4,\ +18,\ -16,\ -36,\ +36$
4^321^2	$-1,\ +4,\ -4,\ +11,\ -9,\ +4,\ -6,\ -10,\ +24,\ +5,\ -7,\ -9,\ +8,\ +1,\ -7,\ +15,\ +16,\ +16,\ -62,\ -24,\ +2,\ -40,\ +45,\ +82,\ +89,\ -72$
43^21^2	$-2,\ +2,\ -1,\ +3,\ +5,\ -9,\ -5,\ +12,\ -1,\ +3,\ 0,\ +2,\ -20,\ -4,\ +15,\ -6,\ +34,\ -20,\ -39,\ +18,\ +2,\ -40,\ -9,\ +62,\ +89,\ -72$
43^321	$1,\ -4,\ -3,\ +6,\ +5,\ 0,\ -17,\ +24,\ -3,\ +6,\ +9,\ -8,\ -1,\ -19,\ +3,\ +24,\ +16,\ -36,\ -51,\ 0,\ +4,\ -16,\ +9,\ +111,\ -144$
42^4	$4,\ 1,\ -2,\ +2,\ -2,\ +2,\ -2,\ +4,\ 0,\ -4,\ 0,\ +4,\ 0,\ -4,\ -2,\ -6,\ +12,\ -12,\ +12,\ -8,\ +12,\ -12$
4^41^4	$1,\ -3,\ +3,\ -1,\ +2,\ +1,\ -8,\ +3,\ -3,\ -3,\ +1,\ -5,\ -1,\ -3,\ +23,\ +8,\ +3,\ +5,\ +6,\ -21,\ -15,\ -8,\ +30$
43	

15120	9030	22360	37800	54600	91140	200	930	2520	12600	21840	31920	54040	90720	360	200	400	600	8960	0160	151300	800	920	9320	2490	2000	8400
124-7400	831-600	166-3200	249-4800	332-6400	498-9600	997-9200	1995-8400	369-600	110-8800	166-3200	221-7600	333-6400	498-9600	665-2800	997-3600	997-9200	1330-5600	1995-8400	3991-6800	7983-3600	748-4400	151-200	369-600	470-400	110-8800	369-600
90	24	108	204	240	468	930	2520	24	108	204	256	480	1128	24	108	200	400	600	8960	800	920	9320	2490	2000	8400	8400
529-200	340-200	705-600	107-7300	143-6400	219-2400	446-0720	907-2480	151-200	470-400	718-200	957-600	146-1600	223-0200	282-360	642-320	453-6000	604-8000	907-200	1161-2160	1814-4000	1496-9800	687-7400	320-0400	691-1600	650-6800	369-600
224-280	138-180	298-200	463-680	616-560	958-2400	198-0720	409-2480	254-0160	470-400	310-800	418-280	642-320	997-920	297-7200	206-1360	453-1360	559-440	415-800	550-3680	408-2400	323-460	148-240	320-2300	660-6080	310-6080	320-5200
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16800	9060	9180	15120	26130	25120	54040	21840	31920	5280	17660	23040	30320	17404	68040	30858	52020	40320	87660	550-9560	14400	40320	30858	20470	6660	1406-1600	2857-3860
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488	144	540	1008	240	1080	2530	108	108	204	204	256	480	360	24	108	200	400	8960	151300	800	920	9320	2490	2000	8400	8400

60	30	103	207	321	618	1713	4536	1153	2277	3192	6177	1620	4500	13012	31221	79632	200340
88	24	124	291	390	872	2459	6888	189	390	798	1153		5940	15840	41310		498960
828	344	1056	1875	2390	4248	9680	21840	258	216	488	1048		6660	15540	36336	85140	1108800
1080	360	1332	2505	3060	5820	13590	31920	150	488	1048	798						
438	144	540	1008	1230	2520	5400	12600	117	390	828	451	840	1860	4440	10656	25680	62160
90	24	108	204	240	468	1080	2520	108	204	256	480	936	1128	2220	2640	5280	12600
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6420	3420	9110	15975	22425	38670	91725	214200	1080	5400	9630	13590	23720	40980	56990	97300	134400	226400
2694	1294	3711	6678	9140	16301	38670	91140	2520	2328	4248	10428	18504	25120	44088	59640	103920	149920
1500	560	1905	3705	4800	9140	54600	37800	1080	1230	2390	5840	10860	14250	26180	34600	62600	70300
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610	204	763	1506	1905	3711	1950	9030	204	540	1332	2578	4900	6284	11841	15270	28530	73080
201	36	204	498	560	1294	1280	15120	108	144	344	844	1812	2160	4524	5620	11440	51240
456	201	610	1116	1500	2694	690	8680	24	438	828	2026	3768	4900	9054	11760	21600	31360
244	80	314	599	755	1466	1680	4900	90	237	467	1140	2210	2779	5361	6760	12980	75520
126	24	147	189	360	781		3360	24	132	264	644	1296	1570	3174	3840	7780	19110
76	12	78	117	210	488	1075	1680	66	153	376	817	954	2046	2460	5180	13160
48	12	58	117	140	284	29400	11844	58	117	286	584	699	1434	1710	3525	8680
840	630	1715	3150	5480	9660	70560	6888	630	1260	4620	8400	14000	24570	38640	66360	70520
360	220	675	1305	2135	3970			270	570	2020	3840	6070	11130	16800	30130
150	78	264	528	828	1593			117	258	886	1048	1442	2976	4056	8132

+2	+2	+8	−4	−2	−3	−5	−3
360	220	675	1305	2135	3970	11095	29400
150	78	264	528	828	1593	4446	11844
88	24	124	291	390	872	2489	6888
60	30	103	207	321	618	1713	4536
	20	60	120	335	630	2355	7560
28	6	22	48	120	245	910	2940
34	6	30	68	80	172	440	1120
13	8	47	110	145	327	916	2548
		9	36	10	20	135	600
	2	8	18	30	106	320	966
12		3	10	43	90	331	1064
12	4	12	31	15	47	174	588
6		18	39	3	7	46	204
4		7	18	30	80	210	560
1		3	11	55	114	315	840
			2	20	53	150	420
−2	−3	6	6	10	32	95	280
	1	3	3	16	6	20	70
	1	1	5	14	30	108	336
		1	2	4	6	51	168
			1	2	13	15	56
				1	6	13	56
					28	6	28

270	117	51		30	27	6	18
570	258	150	117	68	68	24	39
1050	444	216	189	90	36	72	96
2020	886	488	390	180	78	168	216
3840	1752	1048	798	360	168	368	476
6070	2640	1442	1153	930	392	424	632
11130	5037	2976	2277	1740	786	924	1344
16800	7308	4056	3192	3440	1440	1090	1762
30130	13656	8132	6177	6210	2780	2840	3670
78120	35658	21644	16212	19320	8680	5950	9793
560		90720	56448	41580	55440	25200	15120
7200	3420	2160	1620	720	360	795	1020
20220	9540	5940	4500	3240	1560	1990	2800
53640	25380	15840	12012	11160	5316	5022	7500
070	65412	41310	31221	339930	16200	12720	19722
808	79632	51128	95750	46200	322270	2970	7737

	4^24	4^214	43^21^3	42^31^2	431^5	42^14	421^6	41^8	3^4	3^321	3^22^3	31^3	$3^22^21^2$	32^41	3^221^4	32^31^3	321^6	32^15	321^7	31^9	2^6	2^51^2	2^41^4	2^31^6	2^21^8	21^{10}	1^{12}
4^24	1	−12	+12	−8	+12	−12	−8	+12	+12	−6	−2	−4	+12	−4	0	+4	0	−4	0	+4	0	−4	−2	+2	−2	+2	−2
4^214		1	−8	−15	−21	+6	+5	+3	+8	+23	+8	+3	+5	+6	−1	−5	+1	−3	−3	+1	+2	−1	+3	−3		+30	+240
43^21^3			1	−12	−100	−108	+240	+68	+73	−27	+5	+48	−33	−17	−12	+7	+7	−3	+4	−3					+73	+240	+120
42^31^2				1	−76	+10	−39	+16	+6	−15	+24	−39	−18	+8	+3	−1	+8	+9	−16	−4	−2					+120	−72
431^5					1	−72	+32	+18	−8	−5	17	−22	+5	0	+5	−1	+8	−2	−1		+5					−72	−180
42^14						1	−180	+70	+70	+45	−20	+5	+54	+12	−1	−20	+5	−6	+20	+2						−180	+84
421^6							1	+84	−18	−14	−21	+4	+23	+8	−1	−2	−8	+6		+28						+84	−12
41^8								1	−12	+1	+2	+3	−3	−1		+56	8	28	56	168	336	70	280			−12	+3
3^4									1	+3	−3	+6	−3	−3	+3	+3	−3	+3	+3							+3	−48
3^321										1	+4	−3	+19	+13	−12	−27	−12	+13	+16	+28	+37	−48		3			−24
3^22^3											1	−3	+9	+6	−5	−11	−3	+8	+24	−24	−6	+8	−3	6	11	2	+40
31^3												1	+2	+3	+6	−11	−5	+8	+17	−30	−18	+40	−2	+2	6	3	+180
$3^22^21^2$													1	+9	+9	−4	−5	+54	+63	−45	−114	+180	−4	+9	2	8	+60
32^41														1	−3	+5	−7	+9	−6	−4	−6	+20	−49	+60	9	22	−180
3^221^4															1	−3	−6	+13	−45	−4	−36	+70	+70	−180	12	36	−240
32^31^3																1	−5	+14	−30	−21	0	+130	−240	+24	60	93	+42
321^6																	1	−2	−1	+9	+2	+3	−17	−9	+42	+252	12
32^15																		1	−7	+27	−4	+18	−32	−87	+252	5	40
321^7																			1	−9	+16	+19	−96	−3	3	38	168
31^9																				1	+12	−1	−2	9	28	189	1080
2^6																					1	−2	+2	−2	+2	−2	+2
2^51^2																						1	−4	+9	−16	+25	−36
2^41^4																							1	−6	+20	−50	+105
2^31^6																								1	−8	+35	−112
2^21^8																									1	−10	+54
21^{10}																										1	−12
1^{12}																											1

If instead of arranging the partitions in the natural order we had arranged them as follows: a b_1, b_2, b_3 ... c_1, c_2, c_3 ... where a is the partition of one part, those in the group b the partitions of two parts in dictionary order, etc., the same conditions would obtain, and it is upon this latter arrangement that the accompanying table is based.

Let us now separate the partitions, which are, say, $n - 1$ in number, into pairs consisting of a partition and its conjugate. Designate the prior partition of a pair by b_ν and the other by $b_{n-\nu}$. Call the unpaired (self-conjugate) partitions c_1, c_2 ... c_k.

If now we arrange the partitions b_ν in the order in which they occur in the first arrangement, and place after them the c_1, c_2 ... c_k, I say the order so obtained will give the desired form to the table.

We have by eq. (2)

$$(\nu.n - \pi) = 0 \quad \pi > \nu,$$

since the conjugate of $b_{n-\pi}$, *i. e.* b_π is subsequent in natural order to b_ν. The order of the c's is arbitrary, since by (4) the P of a self-conjugate partition can contain the ϕ of no other self-conjugate partition. If then we arrange the ϕ functions across the top of our table in the order last named, and the P functions down the side in the same order, there can be no coefficients above the sinister diagonal except in the case of the self-conjugates. The coefficients on the sinister diagonal will be units since (Cayley, *l. c.*) $(\nu.n - \nu) = 1$. The self-conjugates will, for the same reason, have unit coefficients on the principal diagonal, and these unit coefficients will be symmetrically placed with reference to the sinister diagonal. Since $_-(\lambda\mu) = (\mu\lambda)$, the coefficients similarly placed with reference to the principal diagonal will be equal, *i. e.* the table will be symmetrical.

It is evident that the table of the values of the ϕ's in terms of the P's will be similar in form except that the coefficients will occupy the part *above* the sinister diagonal. This diagonal will consist of units as before, and if we agree to consider these units as belonging to both tables we may write both tables in the same square.

The accompanying tables of the twelfthic were published in a different form in a former number of this Journal, but it is thought that the new arrangement is of enough interest to warrant reprinting. These tables have also been calculated by M. Řehořovsky, and appeared in the Transactions of the Royal Academy of Vienna.